A Preliminary Design of a $\mu^+\mu^-$ Collider Ring

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Abstract

A very preliminary design of a $\mu^+\mu^-$ collider ring is presented briefly. It consists of the design concepts:

- 2.5π unit cell
- local chromaticity correction
- quasi-isochronous ring
- improvement of dynamic aperture using octupoles and decapoles.

The first two concepts have been verified in the KEKB B–Factory successfully.

Machine Parameters

A storage ring has been designed to satisfy the requirements on a $\mu^+\mu^-$ collider given by R. B. Palmer *et al.* (BNL–62740). This ring has parameters which will be at the extreme of possible storage ring colliders, as shown in Table 1:

The Unit Cell

Issues to be considered in designing the unit lattice are:

- The circumference must be minimized for a given field of a bend.
- The ring must be quasi-isochronous, *i.e.*, $\alpha_p \lesssim 10^{-5}$. Since the injected beam has $\sigma_z = 3$ mm and σ_δ =0.2%, if there is no rf, the momentum compaction must be less than $\sigma_z/(Cn_{\rm eff}\sigma_\delta) \sim 2.5\times10^{-7}$, which looks too difficult, because of tolerances on the controlability, nonlinear terms, etc. Higher synchrotron tune requires more rf, which is source of instability. Smaller synchrotron tune is difficult due to the nonlinear momentum compactions.

[1]Assuming $f_{\rm RF} = 500$ MHz.

Table 1: Parameters of this design comparing with BNL–62740.

		This design	BNL–62740	
Beam energy	E	2		TeV
Lorentz factor	γ	18900		
Inj. emittances	$\gamma\varepsilon_{x,y}$	50	50	μm
Inj. mom. spread	σ_δ	0.2	0.2	%
Inj. bunch length	σ_z	3	3	mm
Bending field	B_D	10	9	T
Circumference	C	~ 6	7	km
Effective turns	n_{eff}	~ 1000	~ 900	
Beta at IP	β_x^*/β_y^*	3/3	3/3	mm
IP free length	ℓ^*	6	6.5	m
IP beam sizes	σ_x^*/σ_y^*	2.8/2.8	2.8/2.8	μm
IP quad field	B_0	6.5	6.4	T
IP quad radius	a	200	120	mm
Momen. compact.	α_p	5.4×10^{-6}		
Rf voltage[1]	V_c	2.2		GV
Synch. tune	ν_s	0.003		
Repetition rate	f	–	15	Hz
Muons/bunch	N	–	2×10^{12}	
Bunches/beam	N_B	–	2	
Luminosity	L	–	1×10^{35}	cm^{-2}s^{-1}

- It would be better (not yet confirmed) to have a non-interleaved sextupoles to reduce the nonlinearity of sextupoles.

A 2.5π cell, which has been applied to KEKB, has a capability to satisfy above requirements. Figure 1 shows the unit cell of this design. This unit cell has a momentum compaction $\alpha_p = -2.2 \times 10^{-4}$ which results $\alpha_p = 5.4 \times 10^{-5}$ together with the compaction from the interaction region(IR). Some parameters like number of cells/ring (24 in this design), length of quads, etc. have not been optimized yet.

Quasi-Isochronous Ring

When the linear momentum compaction is small, the higher order momentum compaction limits the size of the rf-bucket.

- The third order term can be controlled by sextupoles with the term $H = k'\eta^3\delta^3/6$. The solution is obtained simultaneously with the chromaticity correction.
- The the fourth order is significant. This is also optimized by sextupoles in this design, but octupoles may be more efficient (not yet tried).

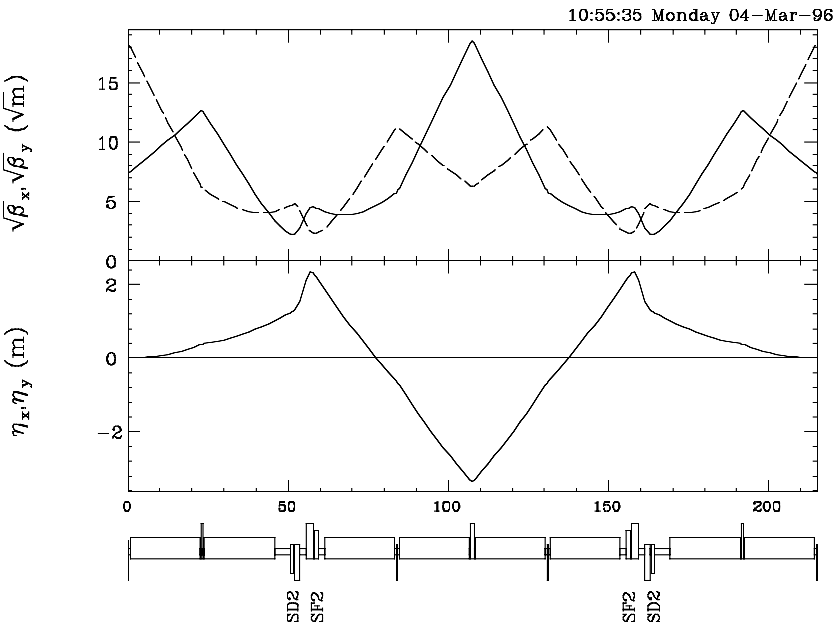

Figure 1: The unit cell of this ring. Sextupole pairs are placed with pseudo $-I$ transformations. The phace advances per cell are 2.5π for both planes. This ring has 24 unit cells.

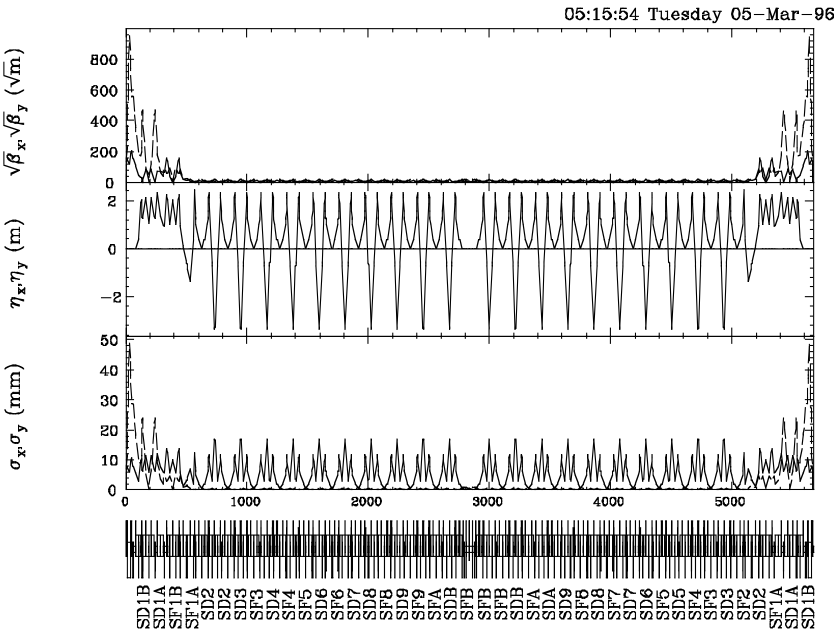

Figure 2: The optics and the injected beam sizes of the ring, from IP to IP. 22 sextupole families are installed.

Figure 3 show $\Delta p/p$ vs. $\Delta p/p/\alpha_p$ (path length) after the optimization. It is seen that the third order term is nearly cancelled and the fourth order is dominant. The corresponding phase space is shown in Fig. 4.

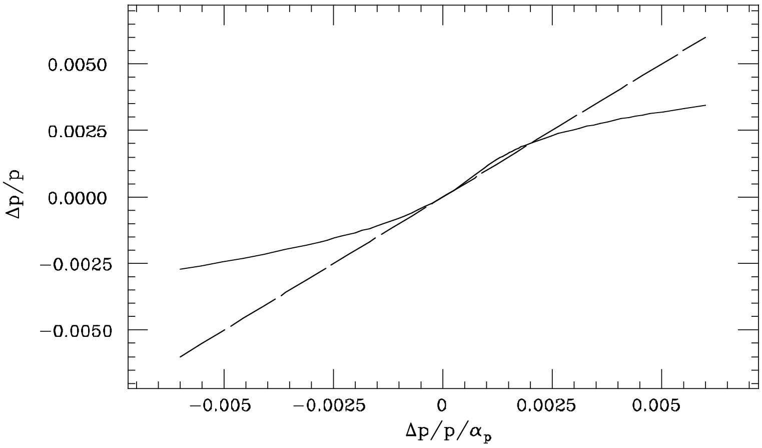

Figure 3: Moentum offset vs. path length.

Chromaticity Correction

Because of the round-beam focusing with very small βs, the chromaticity correction becomes very difficult in this ring. Table 2 compares the chromaticities of this ring with other machines:

	This design	KEKB	JLC-I	
β_x^*/β_y^*	3/3	330/8	10/0.1	mm
ℓ^*	6	1.9	2.5	m
ξ_x/ξ_y (each side)	340/4300	76/270	250/3400	

The basic strategy of the chromaticity correction is the "local correction" with 2 families of sextupoles around IP, each of them consists of a pair of sextupoles connected by $-I$. Since our goal is a round-beam focusing, there are four combinations of the order of the focusing and chromaticity correction planes:

Focus	Correction	Difficulties
IP–y–x	y–x	x-chromaticity correction
IP–y–x	x–y	x-higher order dispersion from sexts.
IP–x–y	x–y	y-chromaticity correction
IP–x–y	y–x	

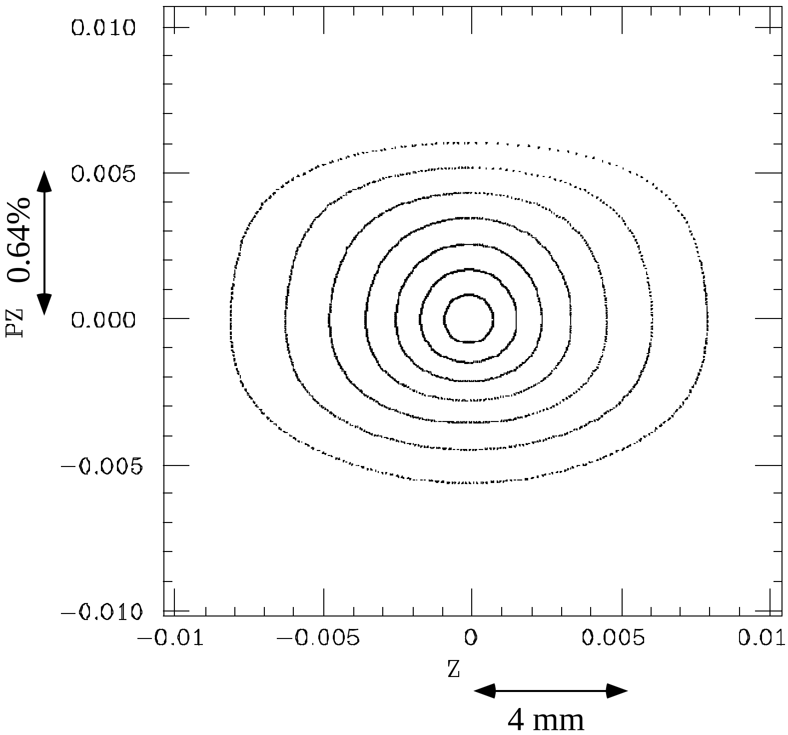

Figure 4: Shape of the longitudinal phase space.

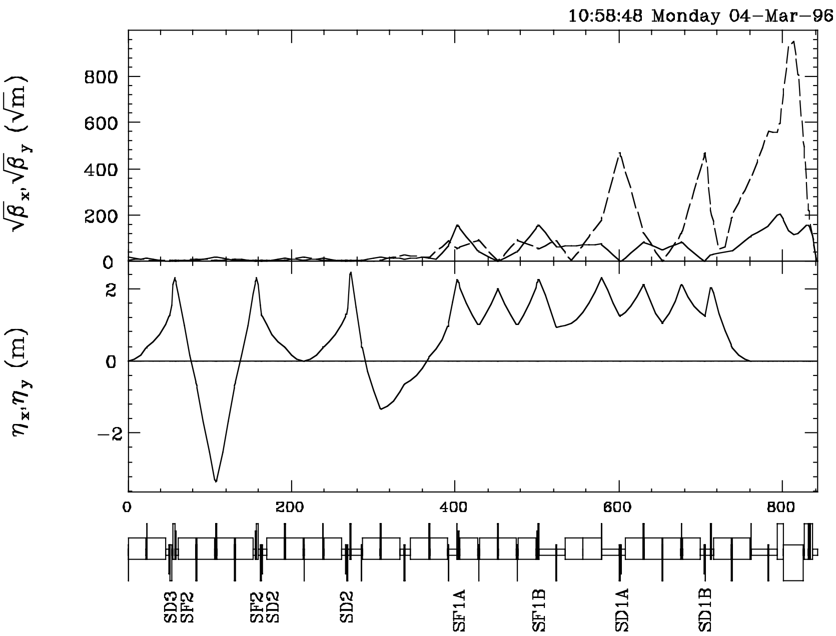

Figure 5: Optics of the local chromaticity correction section

Thus this design applies the last one. The design of optics of the local chromaticity correction is shown in Fig. 5.

This scheme prefers smaller x- and larger y-chromaticities. The design of the final quads are so chosen, as shown in Fig. 6.

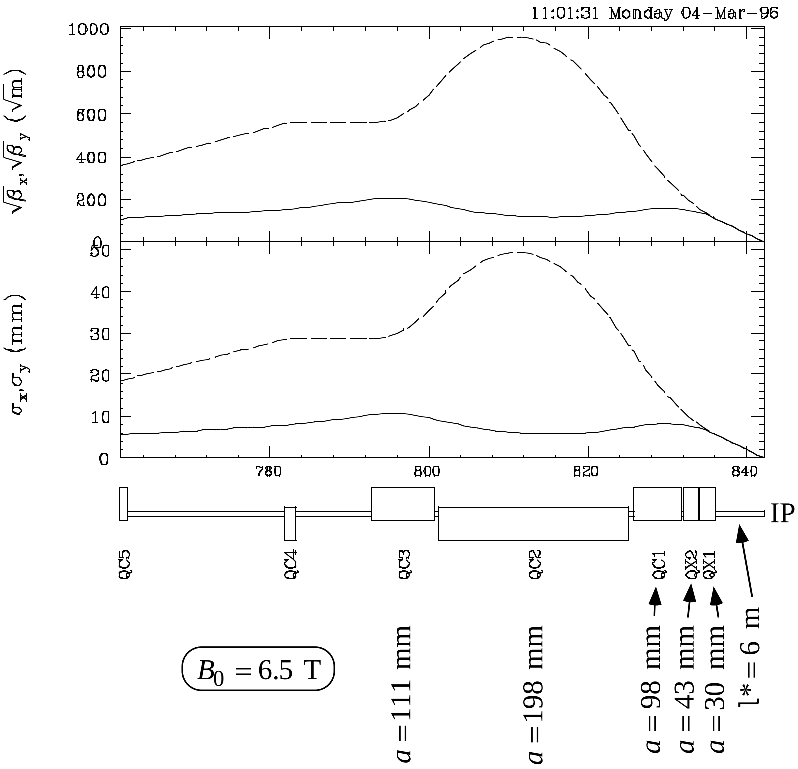

Figure 6: Betas and beam sizes in the final quads. The aperture of final quadrupoles with the pole-tip field of 6.5 T are also shown.

Higher order chromaticities are corrected by 22 families of sextupoles. An optimizations of momentum dependence of tunes, betas at IP and rf for off-momentum orbits and finite-amplitude trajectories and higher-order momentum compactions were done. The optimization involves correction octupoles and decapoles around IP. The resulting momentum dependences of tunes and betas are shown in Fig. 7.

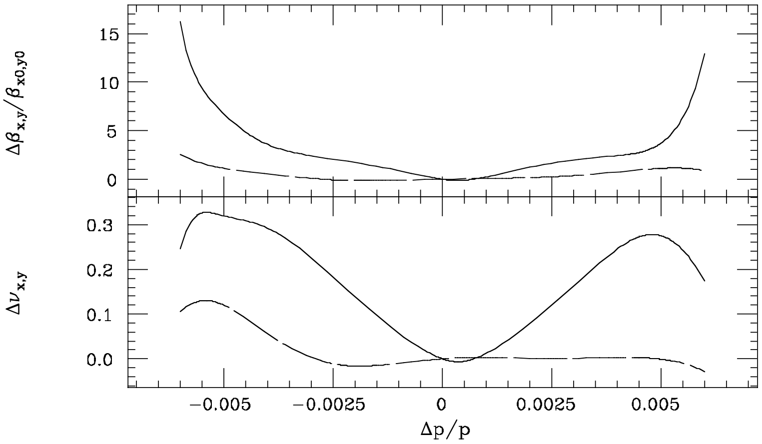

Figure 7: Momentum dependences of tunes and $\beta^*_{x,y}$. The horizontal(solid) plane is more difficult than the vertical(dashed).

Dynamic Aperture

The dynamic aperture of this ring is limited by

- The nonlinear maxwellian fringe of final quads and quads around local correction sextupoles. It is corrected by additional octupole components of final quads. Since this is not a local compensation, their chromatic dependences remain. Decapole components of quads at IR are used to correct the residual.
- The imperfectness of the chromaticity correction.
- Asymmetry of the synchtotron phase space due to $\exp(:\delta^3:)$ term.

The strengths of octupoles and decapoles are determined by "Finite Amplitude Matching", which fits linear optics around trajectories starting IP with an offset $u = (J_x, \phi_x, J_y, \phi_y, 0, \delta)$. 18 trajectories are chosen with amplitudes

$$\begin{aligned}
u &= (8\varepsilon_x, 2n\pi/3, 0, 0, 0, 0), && n = 0..2\\
u &= (0, 0, 8\varepsilon_y, 2n\pi/3, 0, 0), && n = 0..2\\
u &= (0.25\varepsilon_x, (2n+5)\pi/3, 0, 0, 0, 0.002\times m), && n = 0..2, m = -1, 1\\
u &= (0, 0, 0.25\varepsilon_y, (2n+5)\pi/3, 0, 0.002\times m), && n = 0..2, m = -1, 1.
\end{aligned}$$

Figures 8 and 9 show the results of the dynamic aperture with and withoput synchrotron motion, respectively. The effects of octupoles and decapoles look quite strong.

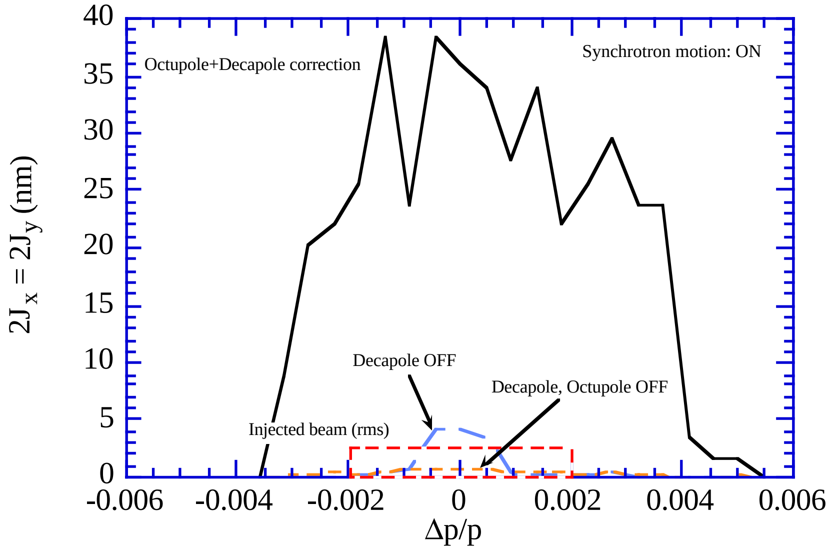

Figure 8: Dynamic aperture of this ring with synchrotron motion. Octupole and decapole corrections improve the aperture drastically.

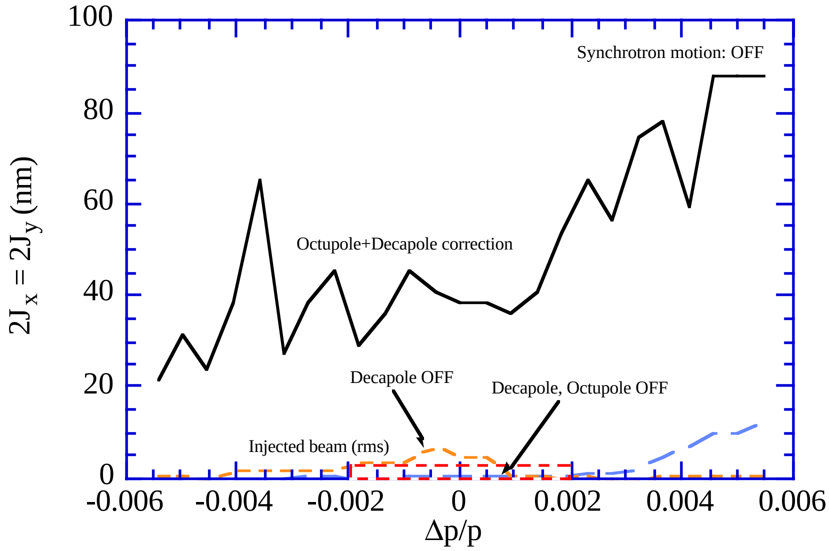

Figure 9: Dynamic aperture of this ring without synchrotron motion. Octupole and decapole corrections improve the aperture drastically.